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Semi-analytical Recursive Algorithms of Convolution Calculations for Digitally Controlled Buck Converter Design

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***Abstract*—Modern power systems have to ensure the quality of power supply in spite of the rapid changes of power consumption. Choosing the appropriate parameters of the transfer function of the digital control circuit of the converter main block is especially important. The most common method of obtaining the discrete transfer function of control circuit is an analog prototype. An important problem is the accurate conversion of the analog transfer function in the discrete transfer function. Inaccurate transformation of the analog transfer function causes significant deterioration of the dynamic parameters of the converter controlled digitally in comparison to the converter controlled by an analog control system prototype. The paper shows how to use semi-analytical recursive algorithms of convolution calculations (SARA) to convert analog transfer function in discrete form. With SARA algorithms it is possible to reduce twice the frequency of the digital circuit of the converter in comparison to the frequency of the converter power stage.**

***Keywords*—digitally controlled Buck converter, SARA, semi-analytical recursive convolution**

I. INTRODUCTION

Because of its high energy efficiency, small dimensions and small weight, Buck converters are widely used in power supplies. The disadvantage of such converters is the sensitivity of the output voltage of the power stage to the change of circuit's operating point. The most common method of obtaining the discrete transfer function of control circuit is a method of using analog prototype. The transfer function of the analogue control system is first determined and then transformed from the Laplace domain field into z domain. In the analog prototype of the Buck converter, should be consider, among others, the delays caused by digital control circuit in feedback loop and parameters of digital PWM [6–12]. Influence of parameters ADC and PWM resolutions on quality of stabilized output voltage have been omitted in the paper [6]. The use of SARA algorithms on the example of a voltage-controlled buck converter is shown. The SARA algorithms can also be used in current control [13–15] or other ways of stabilizing the converter output voltage [16–27]. In addition, the results of research on the converter in which the control system performs calculations every second cycle of the converter have been presented. This makes it possible to use a cheaper control circuit or a more complex numerical algorithm of the converter control system.

II. VOLTAGE MODE CONTROL

The DC/DC converters are mainly used voltage control mode to output voltage stabilization [28–31]. The advantages of this method are: high effectiveness of output voltage stabilizations and accuracy, and also ease of hardware implementation. Small signal model of Buck converter with voltage mode control is presented in Fig. 1.

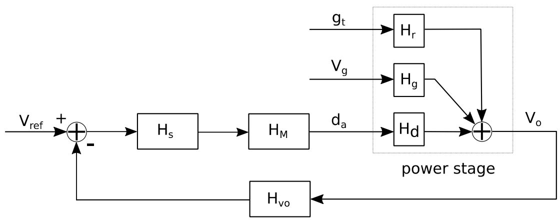

Fig. 1. Small signal equivalent circuit model of Buck converter with voltage control mode.

where

- H_s – transfer function of the control block
- H_M – transfer function of the digital PWM modulator
- H_d – transfer function describing the influences of d_a on v_o
- d_a – small signal PWM duration changes
- H_g – transfer function describing the influences of v_g on v_o
- v_g – small signal voltage supply changes
- H_r – transfer function describing the influences of g_t on v_o
- g_t – small signal output conductance changes
- H_{vo} – equivalent transfer function of the voltage divider, anti-aliasing filter (serial connection)

The small signal model describes the power stage of Buck converter as presented earlier in this paper [32-37]. PWM modulator generates a signal to control power stage of converter. The operating of the PWM modulator is described by the transfer function H_M containing the ideal delay [10, 12, 38–41]:

$$\mathrm{H_M} = e^{-s \cdot t_d} \quad (1)$$

where

t_d – total delay introduced by the digital modulator and converter digital control system

Desired closed-loop characteristics is achieved by shaping open loop transfer function H_{OL}, using Bode plots [28, 30, 31]. The transfer function H_{OL} is given by the equation:

$$\mathrm{H_{OL}} = \mathrm{H_s} \cdot \mathrm{H_M} \cdot \mathrm{H_d} \cdot \mathrm{H_{vo}} \quad (2)$$

Stabilization of the output voltage is effective if following inequality will be satisfied [30, 31, 37]:

$$|\mathrm{H_{OL}}| \gg 1 \quad (3)$$

Using transfer function H_{OL} to find the transfer function of control circuit H_s it should be considered that inequality (3) can be fulfilled only for frequencies less than ~ 0.5 f_{PWM}. Otherwise, the control circuit would be sensitive to the noise generated by the power stage of converter (switching noise). The value of the frequency f_c for which the equation (4) is satisfied should be as large as possible, and then the control circuit is more responsive to changes in the output voltage. The method used to evaluate stability and shape of the transfer function H_{OL} of the control circuit are given in the papers [30, 31].

$$|\mathrm{H_{OL}}(f_c)| = 1 \quad (4)$$

The second important parameter in shaping the transfer function H_S of control system is the phase margin (PM).

$$\mathrm{PM} = 180° + \varphi(\mathrm{H_{OL}}(\mathrm{f_c})) \quad (5)$$

Converter control system with feedback loop is stable if PM ≥ 0. In practice, the PM value have to be much greater than zero, due to the variation of value of electronic components and the increasing amplitude of the transient state v_o when $\varphi(H_{OL}(f_c))$ tends to -180° [30, 31, 37]. Typically, PM is within the range <40°, 100°> [42–44].

The most commonly used transfer function of the analog prototype of the control system is the transfer function of the type III, which has two zeroes and three poles:

$$\mathrm{H_{s2z3p}} = \mathrm{K_{dc}} \cdot \frac{(\mathrm{s}-\mathrm{z_1})\cdot(\mathrm{s}-\mathrm{z_2})}{(\mathrm{s}-\mathrm{p_1})\cdot(\mathrm{s}-\mathrm{p_2})\cdot(\mathrm{s}-\mathrm{p_3})} \quad (6)$$

All zeros and poles real of H_{s2z3p} are real. Pole p_1 is always placed at zero [28–31, 42, 43, 45–51].

III. Discretization of the Transfer Function with Semi-analytical Recursive Algorithms

The properties of semi-analytical recursive algorithms (SARA) and use them to model the operation of electronic circuits (including converters), are discussed in the papers [53–59]. Also, possibilities of using SARA algorithms to synthesize digital controllers of converters are briefly described in [52]. The response of the converter control system to the output voltage changes in the time domain is determined numerically by convolution calculations of function:

$$y(.) = h(.) * x(.) = \int_0^t h(t-\tau) \cdot x(\tau) d\tau \quad (7)$$

where

y(.) – output signal from the control system,

h(.) – impulse response of the control system,

x(.) – input signal

The convolution (7) can be efficiently determined by the equation

$$y[n] = \Phi_n \cdot y[n-1] + \sum_{r=0}^{R} A_r \cdot x[n-r] \quad (8)$$

where

$$\Phi_n = \exp\{\alpha_n \cdot \Delta) \quad (9)$$

Δ – algorithm step,

R – the order of an algorithm,

x[n] – input signal sample,

y[n] – output signal sample,

for the transfer function of the control system whose impulse response is

$$h(t) = K \cdot \exp(-\alpha \cdot t) \quad (10)$$

The impulse response (10) is for the transfer function of the character control system:

$$\mathrm{H}(s) = \frac{\mathrm{K}}{\mathrm{s}+\alpha} \quad (11)$$

Each transfer functions of control system:

$$\mathrm{H_S}(\mathrm{s}) = \frac{\mathrm{a_0}+\mathrm{a_1}\cdot\mathrm{s}+\mathrm{a_2}\cdot\mathrm{s}^2+\cdots+\mathrm{a_m}\cdot\mathrm{s}^\mathrm{m}}{\mathrm{b_0}+\mathrm{b_1}\cdot\mathrm{s}+\mathrm{b_2}\cdot\mathrm{s}^2+\cdots+\mathrm{b_n}\cdot\mathrm{s}^\mathrm{n}} \quad (12)$$

Which has a real zeros and poles satisfying the condition n ≥ m can be represented as the sum of the transfer functions $\frac{K}{s+\alpha}$. Because the selected transfer function of the type III (6) has only real zeros and poles [28–31, 42, 43, 45–51], than it can be presented in the form of the sum of transfer functions $\frac{K}{s+\alpha}$ and use the convolution algorithms for discretization.

The coefficients of the discrete equation (8) are given in [52, 57–59]. When in equation (11) the coefficient α is equal 0, the coefficients of the SARA algorithm can be obtained by exchanging equation (9) for its developing at point 0 [57]. In the case of algorithm of the order III, this reduces the numerical complexity of calculations. In power electronics there is a rule that the price of the control system cannot be higher than the price of the converter power stage. For this reason, the computational capabilities of the microcontrollers used in the Buck converter control systems are a significant limitation. Therefore, in addition to the accuracy of the analog function mapping of the control system, numerical complexity is also important. Tab. 1 shows the number of multiplication and addition operations needed to implement the analog transfer function of the type III (6) in a digital circuit.

TABLE I
NUMERICAL COMPLEXITY OF CALCULATIONS

Transformation method	multiplication and addition operations
impulse	8
foh	8
tustin (bilinear)	8
zoh	7
matched	7
SARA R=3 *	9
SARA R=2	8
SARA R=1	7

* for the SARA order three (R=3) without exchanging equation (9) for its developing at point 0, number multiplication and addition is equal 11

Impulse and zoh transformations are not used in practice due to the low accuracy of conversion of analogue transmittance into digital. Matched transformation is the most commonly used method [10, 11, 38] due to the lowest numerical complexity and good accuracy of the analog function mapping. Tustin and foh transformations offer slightly better mapping accuracy than matched transformations, but they are more complex numerically (Tab. 1). The greatest numerical complexity has a discrete function obtained by transformation an analog function using the third-order SARA algorithms (Tab. 1). For this reason, in the presently used digitally controlled converters, this algorithm is not widely discussed and applied.

IV. MODELING AND SIMULATION OF BUCK CONVERTER

Simulation of the proposed model was performed in MATLAB/SIMULINK and Simscape. Digitally controlled Buck converters were tested in which discrete transfer functions were obtained using all of the algorithms shown in Tab. 1. The results of the research for the transfer functions were obtained by algorithms: tustin, matched, SARA of the order two and three. For converters in which discrete functions were obtained by tustin and foh methods, the simulation results were almost identical. Likewise, the differences between the SARA algorithms of the order two and three were negligible, Fig. 3, 4 and 5.

The digitally-controlled buck converter – Texas Instruments TMDSC2KWRKSHPKIT Development Board was used to verify the proposed method of finding the transfer function of control circuit. According to the documentation of the Development Board, it was assumed that equivalent circuit parameters are:

$L = 10\ \mu H$, $R_L = 42.4\ m\Omega$, $C = 726\ mF$, $R_C = 40\ m\Omega$, $G = 1/7.5\ S$ (converter load)

$R_{K1ON} = 5\ m\Omega$, $R_{K2ON} = 1.1\ m\Omega$ (drain to source resistance in ON state)

$f_{PWM} = 300$ kHz (PWM frequency), $V_G = 9$ V (power stage supply), $D_A = 0.5$ (PWM duration in steady state)

Design assumptions:

$PM = 45°$, $f_c = 25$ kHz,

$t_d = 0.5/f_{PWM}$ (the delay introduced by the digital control circuit) [9, 10, 12]

Test signals:

step change in the voltage v_g – $0.35 * V_G$

step change in the load g_t – ½ S

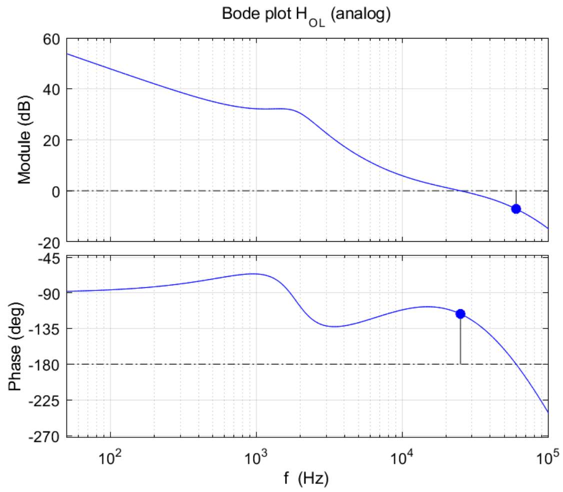

Fig. 2. Bode plot of the analog prototype H_{OL}.

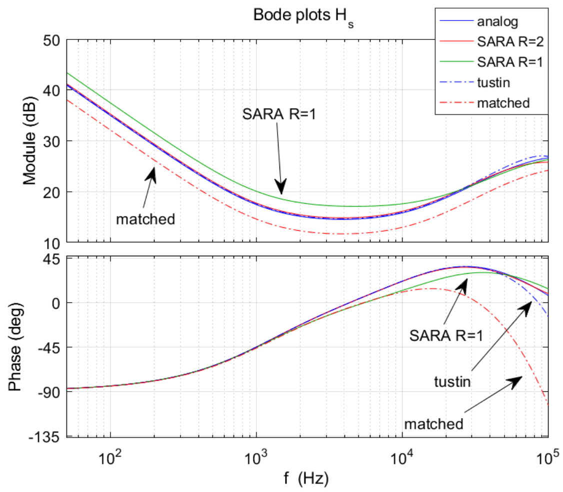

Fig. 3. Bode plot of the transfer function H_S of control system.

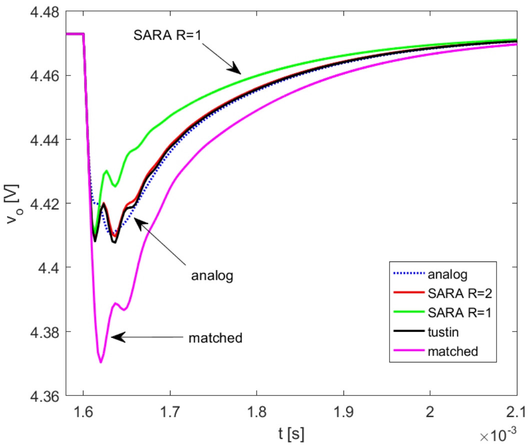

Fig. 4. Voltage v_o changes under the influence of voltage v_g.

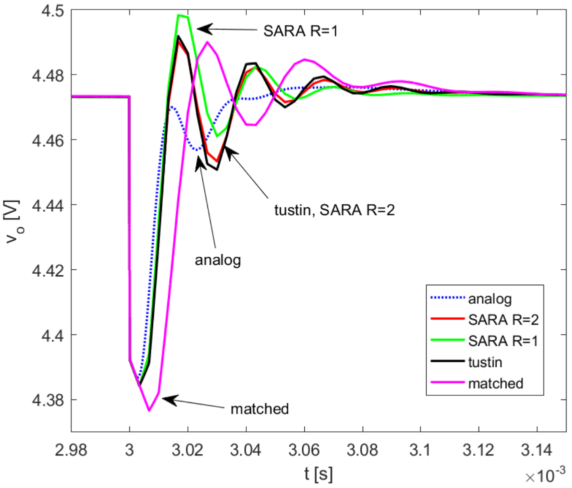

Fig. 5. Voltage v_o changes under the influence of voltage g_t.

V. Reducing the Value of Frequency of the Control System

A significant limitation of the use of converter digital control inverter in practice is the low efficiency of the calculation units. It is unprofitable to use digital control circuits more expensive than the price of the converter power stage itself. This results in two significant design constraints:

- the complexity of the control circuit algorithm cannot be high, so a matched transformation is used.
- the switching frequency of the power stage is limited by the time it takes to make the necessary calculations in each cycle. Therefore, converters controlled in analogue technology can achieve higher operating frequency and better dynamic parameters [9].

Typically, the control system must be able to make calculations during a single duty cycle of the converter. This paper presents the test results for a digitally controlled converter when the control system performs a measurement every second cycle of the converter. In this case, the time needed for calculations by the digital circuit is twice as long as the classical solution. The sampling of the output voltage v_o every second cycle of the converter and the increase of the calculation time of the algorithm to the two cycles forced the reduction of the f_c frequency from 25 kHz (first test, Fig. 2) to 18 kHz (Fig. 6). For higher frequencies f_c oscillations have occurred during the transient state. This is caused by a double reduction of the sampling rate from 300 kHz (f_{PWM}) to 150 kHz (1/2 f_{PWM}).

Bode plot of the analog prototype H_{OL} with reduced sample rate is shown in Fig. 6.

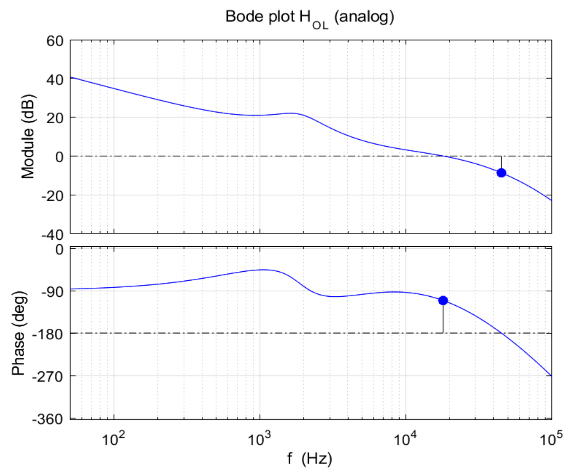

Fig. 6. Bode plot of the analog prototype H_{OL} with reduced sample rate.

Changes in the stabilized output voltage of the converter in time domain under the influence of the change in the supply voltage v_g and the load conductivity g_t are presented in Fig. 7 and 8.

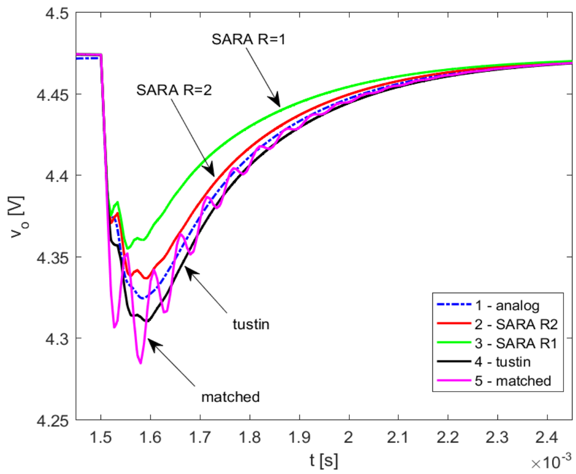

Fig. 7. Voltage v_o change under the influence of voltage v_g step change for different transformation methods from domain *s* to domain *z*

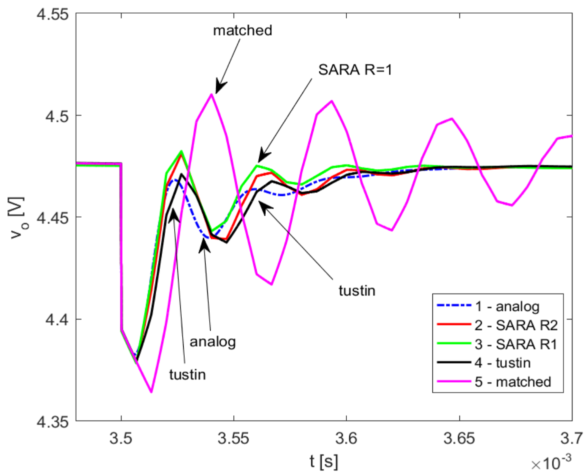

Fig. 8. Voltage v_o change under the influence of voltage g_t step change for different transformation methods from domain *s* to domain *z*

VI. Concluding Remarks

In the paper authors proposed method of using semi-analytical recursive algorithms of convolution calculations to transform transfer function of the control system of the Laplace domain to z domain. Bode plot in Fig. 3 shows how different transformation methods distort the frequency characteristics of the digital control system compared to the analog prototype. This results in a deterioration of the output voltage stabilization parameters, Fig. 4 and Fig. 5. Because of the moderate numerical complexity, the first and second-order SARA algorithms are practically used in converter control system. The first-order SARA algorithm offers a better representation of the analog control function compared to the matched method, Fig. 4 and Fig. 5. Both transformations (firs-order SARA and matched) have the same numerical complexity, Tab. 1. The above mentioned advantages (numerical complexity, accuracy of analog prototype representation) are particularly important in the design of digital control circuits for converters operating with high frequency f_{PWM}. The second-order SARA algorithm offers the same numerical complexity and accuracy of the analog transfer function mapping as the tustin / bilinear or foh method, Tab. 1.The performance of the converter with twice the reduction of the sampling frequency (1/2 f_{PWM}) and twice the computation time (2 / f_{PWM}) show that the use of the semi-analytical SARA first order convolution algorithm can be a very good solution.

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